NONLINEAR CONVERSION OF ELECTROMAGNETIC WAVES INTO ION-SOUND PLASMA OSCILLATIONS

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In [1] the authors considered the self-consistent problem of the generation of ion-sound plasma oscillations by strong plasma oscillations (random phases), where the excitation of ion-sound oscillations is accompanied in the one-dimensional approximation by the appearance, in the plasma oscillation spectrum, of satellites differing from the basic frequency by frequencies of the order of ω_{0i}. There has recently been a rise in interest in this question, since experimental investigations indicate [2] that such excitation does in fact occur. Moreover experimental results are in qualitative agreement with the predictions of one-dimensional theory (even when the magnetic field is not taken into account), as regards the distribution of satellites in the plasma as a function of the ratio of the levels of high- and low-frequency oscillations. The results of [2] led us to institute a more detailed analysis of the question of the development and distribution of satellites when the linear absorption of ion-sound waves, which was neglected in [1] and is of importance under certain circumstances, is taken into account. The appearance of conditions for which this absorption is important is of interest in the general picture, since the mechanism by which weakly damped plasma waves are converted into easily damped ion-sound waves may ensure that fairly large-amplitude plasma oscillations are strongly damped in a time which is much less than the collision time.[1] In this connection the self-consistent problem of the conversion of high-frequency transverse waves $\omega \gg \omega_0$ into ion-sound oscillations as a result of multi-stage conversion (which turns out to be very much more probable than the direct process), when a transverse wave generates a plasma wave [3] on decay, and the plasma wave subsequently generates an ion-sound wave, is treated in what follows. The calculation is carried out taking the absorption of ion-sound waves into account in the one-dimensional approximation.[2]

1. Taking into account the linear absorption of sound waves may be important in certain cases in the process where a longitudinal plasma wave decays to a plasma wave (satellite) and a sound wave. This absorption is associated with electrons moving in phase with the sound wave, and may be taken into account in the equations of [1] for the number of ion-sound waves N^s by means of the addition of the term (see, for example, [4])

$$-\beta_\Lambda N^s, \qquad \beta_\Lambda = \left(\frac{\pi}{2}\frac{m_e}{m_i}\right)^{1/2}\omega_s .$$

The linear absorption of sound alters the picture of a decay process qualitatively when it has an increment γ, if $\beta \gg 1/2\,\gamma N^l$, i.e.,

$$\frac{W^l}{m_e n_0 v_e^2} \ll \frac{8\sqrt{2\pi}}{3}\left(\frac{m_e}{m_i}\right)^{3/2}\frac{k_s \Delta k}{k_0^2}, \tag{1.1}$$

$$\gamma = \frac{\omega_0^2 k_0}{8 m_e n_0 v_e^2}, \quad k_0 = \frac{1}{3\lambda_e}\left(\frac{m_e}{m_i}\right)^{1/2}, \quad W^l = N_1^l(0)\,\omega_0\,\Delta k, \quad \lambda_e = \frac{v_e}{\omega_0}.$$

Here W^l is the energy of the plasma waves at the initial moment, N^l is the number of waves in the spectrum with $k_1 > k_0$ (the necessary condition for decay), the spectral width is $\Delta k_1 \ll 4\,k_0$, and the wave number of the sound waves generated in the decay process is $k_s = 2 \times (k_1 - k_0)$.

When (1.1) is fulfilled the number of ion-sound waves in the decay process rapidly attains a value determined by the equation of absorption and generation

$$\beta_\Lambda N^s = {}^1/_2 N_1^l N_2^l, \qquad N_2^l \equiv N^l(k_2), \qquad k_2 = k_1 - k_s,$$

after which the variation of N^s follows adiabatically from the variation of the number of plasma quanta, N^l, and N^s remains small.

Thus, N^l may be neglected everywhere in comparison with N^s in the nonlinear equations for N^l, and it immediately follows from these that in the final state all the energy is transferred completely to the one last satellite which can not now decay.

To illustrate the course which such a process takes, we shall give the solution of the corresponding nonlinear equations in the case of two satellites

$$N_0(t) = N_0(0)\left(\frac{N_1(0)N_2(0)}{N_0^2(0)}e^{N_0(0)\gamma t}+1\right)\left(\frac{N_1(0)}{N_0(0)}e^{N_0(0)\gamma t}+1\right)^{-1},$$

$$N_1(t) = N_1(0)e^{N_0(0)\gamma t}\left[\left(\frac{N_1(0)}{N_0(0)}e^{N_0(0)\gamma t}+1\right)\times\left(\frac{N_1(0)N_2(0)}{N_0^2(0)}e^{N_0(0)\gamma t}+1\right)\right]^{-1},$$

$$N_2(t) = N_2(0)\left(\frac{N_1(0)}{N_0(0)}e^{N_0(0)\gamma t}+1\right)\left(\frac{N_1(0)N_2(0)}{N_0^2(0)}e^{N_0(0)\gamma t}+1\right)^{-1}. \tag{1.2}$$

We note that the characteristic time (increment) of the decay process does not change for the condition of strong damping, but that only the distribution of energy over the satellites changes. When the inequality which is the inverse of (1.1) is fulfilled the results of [1] follow.

2. We shall consider in greater detail the decay process when there is a greater quantity n − 1 of red satellites in the conditions given by the inverse inequality (1.1). The equation for the satellite with index i and those of its two associated lines of ion-sound waves corresponding to decay and fusion, allow us to obtain for the quasi-equilibrium states

$$N_i = \frac{N_i^l(\infty)}{N_1^l(0)}, \qquad N_2 = \frac{N_1(1-N_1)}{1-3N_1},$$

$$N_{i+1} = \frac{N_i(N_i - 3N_{i-1})}{3N_i - 5N_{i-1}} \qquad (i \geqslant 2). \tag{2.1}$$

Here N_1 is determined by the equation

$$1 = N_1 + \frac{N_1(1-N_1)}{1-3N_1} + \sum_{i=3}^{n}\frac{N_{i-1}(N_{i-1}-3N_{i-2})}{3N_{i-1}-5N_{i-2}}. \tag{2.2}$$

For example, we have for the two satellites of (2.1) $N_1 = 2/14$, $N_2 = 3/14$, $N_3 = 9/14$. For the five satellites $N_1 \approx 0.060$, $N_2 = 0.075$, $N_3 = 0.085$, $N_4 = 0.110$, $N_5 = 0.165$, $N_6 = 0.505$. It is thus clear that the maximal number of longitudinal wave quanta will correspond to the last two or three "leading" satellites, the "front" propagating in k-space. The "last" satellites (the "front") are followed by the "tail" of low-intensity "initial" satellites, the number of quanta in each of which gradually decreases. The velocity of propagation of the "front" or of the "maximal" satellite in k-space is, in order of magnitude,

$$\frac{\Delta k}{\Delta t} \sim \frac{4k_0}{\tau} = 3k_0\gamma N_1^l(0). \tag{2.3}$$

This velocity decreases as the intensities of the "leading" satellites decrease. This process ceases when the last "leading" satellite reaches k_0, where the entire energy of the waves will consequently accumulate. When the absorption of ion-sound waves is taken into account the picture of the process is even more one in which the energy is transferred completely to the "leading" satellite and the levels of the "tail" satellites fall to zero.

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[1] We are, of course, speaking about oscillations whose Landau damping is negligibly small, $v_\Phi^l \gg v_e$.

[2] The part played by the presence of more than one dimension in the decay of transverse waves into plasma waves is discussed in [3] and leads to the criterion $\Delta\theta \ll (\omega_0/\omega)^{3/2}$.

3. It should be noted that the decay of the plasma wave spectrum in an unbounded plasma is also of interest as regards the more complex process in which this spectrum is generated. The multistage conversion of high-frequency transverse waves ($\omega \gg \omega_0$) to sound waves[1] is an example of such a process. This process is possible in a nonisothermal plasma on condition that $v_e \gg c\sqrt{m_e/9m_i}$. We make use of the one-dimensional equations of [3] for the decays of transverse waves. Relativistic plasma waves with $k^l = \omega_0/c$ are generated in such a decay. If the level of N_1^l generated is high enough so that the damping of ion sound may be neglected and the ratio of the decay increments $l \to s + l'$ and $t \to l + t'$

$$\frac{\gamma}{\alpha} = \left(\frac{\omega}{\omega_0}\right)^2 \frac{8\pi^2}{3} \left(\frac{m_e}{m_i}\right)^{1/2} \frac{c^3}{v_e^3} \gg 1, \qquad (3.1)$$

then for the initial problem we easily obtain that the two-stage decay $t \to l + t' \to s + l' + t'$ proceeds with an increment four times less than the decay increment $t \to l + t'$ [3], if we confine ourselves to one plasma satellite.

This decrease of increment is explained by the fact that at every instant of the slow decay process $t \to l + t'$, the system of lines is in a quasi-stationary state relative to the fast decay process $l \to l' + s$, with a relative accuracy of α/γ, where only 1/4 of the initial quantity of quanta remains in the fundamental line in the quasi-stationary state, and 3/4 in the longitudinal satellite.

The presence of a large number of longitudinal satellites slows down the multi-stage decay under consideration even more. Condition (3.1) holds in the most interesting case when $\omega_0/\omega \lesssim 10^{-2}$ in fact (considering that $v_e > c\,(m_e/9m_i)^{1/2}$).

On the other hand, in the less interesting case

$$\frac{8\pi^2}{3} \left(\frac{m_e}{m_i}\right)^{1/2} \frac{c^3}{v_e^3} < \frac{\omega_0^2}{\omega^2} \ll 1,$$

$t \to l + t'$, the decay proceeds faster and further decays $l \to s + l'$ have no effect on the rate of the first process.

In the case of the two-stage decay process when ion-sound damping is important from condition (1.1) the decay process which at first proceeds according to the law (for example, for a number N_2^t of waves in the initial transverse satellite)

$$N_2^t = N_1^t(0) - \sqrt{N_1^{t^2}(0) - 2N_1^t(0)\,N_2^l(0)\exp\left(N_s^t(0)\,\beta t\right)},$$

$$\beta = \alpha\left(\frac{\omega}{\omega_0}\right)^3, \qquad (3.2)$$

to fairly small values of N_2^t, determined by the equation

$$N_2^t = \frac{\alpha}{\beta} N_2^l(0) \left(\frac{2N_2^t(0) - N_1^t(0)}{2N_2^t - N_1^t(0)}\right)^{\gamma/2\alpha}, \qquad (3.3)$$

and then virtually proceeds no further, since for the corresponding level of the second satellite N_2^l, all the first longitudinal satellite quanta N_1^l which are generated decay practically immediately.

4. We shall dwell briefly on the results of our consideration of the quasi-stationary one-dimensional boundary problems, which is of some interest since a level of longitudinal waves in the plasma sufficient to cause the appearance of nonlinear effects may be created by any sources on the boundary, as for example in [2].

We assume that for $x = 0$ a flux of longitudinal waves enters the plasma. Using the same equations [1, 3] with the change of operator $\partial(\ldots)/\partial t \to V\partial(\ldots)/\partial x$ (where $V = \partial\omega/\partial k$ is the group velocity), we obtain the result that the characteristic decay length in the case of one plasma satellite is of the order

$$x_0 = \frac{3v_e\lambda_e k_0}{2\gamma N_1^l(0)} = 12\lambda_e^2 \Delta k \frac{m_e n_0 v_e^2}{\langle W^l \rangle}. \qquad (4.1)$$

[1]A similar problem with fixed phase is considered in [5].

By way of illustrating this case, we give some solutions of the non-linear equations determining the spatial distribution of the wave number $N^l(x)$.

We have, from the condition (1.1) for strong damping of ion sound:

a) If $k_2 < 0$ the quanta of the satellite fly backwards, and the quanta of the fundamental line forward. We should have $\partial N^l/\partial x = 0$ at infinity in the quasi-stationary case and consequently $N_1^l N_2^l = 0$, which is reasonable for $N_2^l = 0$, $N_1^l \neq 0$, for example. Thus $N_1^l(0)$ and $N_1(\infty)$ are given. The solution has the form

$$k_1 N_1^l + k_2 N_2^l = k_1 N_1^l(\infty),$$

$$N_1^l(x) = N_1^l(\infty)\left\{1 - \left(1 - \frac{N_1^l(0)}{N_1^l(0)}\right)\exp\frac{\gamma N_1^l(\infty)\,x}{3\lambda_e v_e k_2}\right\}. \qquad (4.2)$$

b) If $k_2 > 0$ the quanta of the satellite fly forward $N_1^l(0) \neq 0$, "seeding" $N_2^l(0) \ll N_1^l(0)$, then

$$N_1^l(x) = N_1^l(0)\left(1 + \frac{k_1 N_1^l(0)}{k_2 N_2^l(0)}\right)\exp\left(-\frac{\gamma N_1^l(0)\,x}{3v_e\lambda_e k_2}\right) \times$$
$$\times\left[1 + \frac{k_1 N_1^l(0)}{k_2 N_2^l(0)}\exp\left(-\frac{\gamma N_1^l(0)\,x}{3v_e\lambda_e k_2}\right)\right]^{-1}. \qquad (4.3)$$

When damping is unimportant we have:

a) If $k_2 < 0$ and $N^s(0) = 0$, $N_1^l(0) \neq 0$, for example, are given, the total flux $k_1 N_1^l(0) + k_2 N_2^l(0) = 0$, then

$$N^s(x) = \frac{k_1}{2k_0}\left(N_1^l(0) - N_1^l(x)\right),$$

$$N^l(x) = N_1^l(0)\left[2 - \exp\left(\tfrac{1}{2}N_1^l(0)\gamma^* x\right)\right]^{-1}, \quad \gamma^* = \frac{\omega_0^3 k_0}{12 m_e n_0 v_e^4 k_2}. \qquad (4.4)$$

b) If $k_2 > 0$ and $N^s(0) = 0$, $N^l(0) \neq 0$, the "seeding" $\Delta \sim N_2^l(0) \ll$ $\ll N_1^l(0)$, then

$$N_1^l(x) = \frac{N_1^l(0) + \Delta + \dfrac{k_1\Delta}{4k_0 - k_1}\exp\left(\gamma^* N_1^l(0)\,x\right)}{1 + \dfrac{4k_0\Delta}{(4k_0 - k_1)\,N_1^l(0)}\exp\left(\gamma^* N_1^l(0)\,x\right)},$$

$$\left(\gamma^* = \frac{(4k_0 - k_1)\,\gamma}{6k_0 v_e \lambda_e k_2}\right). \qquad (4.5)$$

We stress that the length of the nonlinear transfer x_0 depends only feebly on the damping. In considering multi-stage decay $t \to t' + l \to t' + l' + s$, Section 3, as a function of the coordinate in the quasi-stationary case for $k_2 < 0$ the length of the nonlinear transfer increases $(k_1 + 2k_0)/k_1$ times, and for $k_2 > 0$ it increases $4k_0/k_1$ times compared with the transfer length for single-stage decay $t \to t' + l$.

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